Quantum Teamwork for Unconditional Multiparty Communication with Gaussian States

Jing Zhang,[1,*] Gerardo Adesso,[2] Changde Xie,[1] and Kunchi Peng[1]

[1]*State Key Laboratory of Quantum Optics and Quantum Optics Devices, Institute of Opto-Electronics, Shanxi University,
Taiyuan 030006, People's Republic of China*
[2]*School of Mathematical Sciences, University of Nottingham, University Park, Nottingham NG7 2RD, United Kingdom*
(Received 1 January 2009; published 11 August 2009)

We demonstrate the capability of continuous variable Gaussian states to communicate multipartite quantum information. A quantum teamwork protocol is presented according to which an arbitrary possibly entangled multimode state can be faithfully teleported between two teams each comprising many cooperative users. We prove that N-mode Gaussian weighted graph states exist for arbitrary N that enable unconditional quantum teamwork implementations for any arrangement of the teams. These perfect continuous variable maximally multipartite entangled resources are typical among pure Gaussian states and are unaffected by the entanglement frustration occurring in multiqubit states.

Introduction.—Information technology is hectically evolving towards the quantum scale. The discovery that entanglement, apart from being a fascinating founding feature of quantum theory, can be employed as a resource for novel or enhanced processing, manipulation, and distribution of information [1] has spurred enormous theoretical and experimental progress. An appealing perspective for future mainstream quantum information technology is to be implemented over a "quantum internet" [2] architected as a network where light buses and atomic storage units are interfaced to enable secure information transfer and quantum computation. The main routes towards the quantum internet are relying on either discrete variable interfaces between single photons and single trapped atoms, which are physical representations of qubits, or on continuous variable (CV) interfaces between light modes and mesoscopic atomic ensembles, which are physical realizations of bosonic modes. Historically, most of the concepts of quantum information and computation have been first developed in the qubit regime and, more recently, generalized to a CV scenario where very promising perspectives have been acknowledged concerning both general theoretical insights and experimental realizations [3], due to the relative simplicity and high efficiency in the generation, manipulation, and detection of special classes of CV states, Gaussian states [4]. However, the two scenarios for quantum technological implementations have mostly progressed in parallel, with no truly radical instance in which the performance of one approach is demonstrated to be unique and unmatched by the other.

Here we introduce a general multipartite communication primitive—the "quantum teamwork"—that cannot be implemented perfectly with N-qubit resources beyond the threshold $N \geq 8$. On the contrary, we identify families of Gaussian multipartite CV entangled states of an arbitrary number of modes which remarkably enable unconditional quantum teamwork implementations over quantum networks. These CV resources, belonging to the class of Gaussian weighted graph states [5–7], are shown to be typical in the space of pure Gaussian states and are ideally unaffected by the kind of entanglement frustration that arises in low-dimensional many-body systems [8]. Our results demonstrate that at a fundamental level CV entanglement offers enhanced perspectives for quantum technology.

Quantum teamwork.—A quantum police department is investigating malicious activities over the quantum internet [2]. A briefing is issued to N agents in the form of an entangled N-party quantum state $\varrho_N^{(0)}$ distributed by the central station, possibly supplemented with classically broadcast information. The chief publicly orders the agents to split in two teams of K and $N - K$ members, respectively, where we shall assume $K \leq N - K$ here and in the following. The parties carry their share of the entangled resource, and the two teams independently proceed in the investigation. They gather progress encoded in generally entangled states $|\Phi\rangle$ of up to K subsystems, supplied by independent informants. Suppose team A wishes to communicate some undisclosed information to team B. To do so, the members of each team must act cooperatively on their respective blocks of the original entangled system, in such a way that the distributed resource $\varrho_N^{(0)}$ is converted into a perfect K-user quantum teleportation channel, i.e., a tensor product of K maximally entangled states of two subsystems, each belonging to one of the teams. One team can then transfer with perfect fidelity the K-party state $|\Phi\rangle$, containing the investigation progress, to the other team which may process or store it for further inspection. The teams then meet up for a new briefing, another entangled resource $\varrho_N^{(1)}$ is redistributed, and possibly a different configuration of the team squads is ordered for the subsequent stage of the investigation. And the process continues for n steps until the case is closed.

We have just introduced an instance of *quantum "teamwork."* It is a very powerful and practical primitive that generalizes conventional quantum teleportation [9,10] and

multipartite teleportation networks [11] and promises to be of many uses in quantum technology implementations. Crucial for the feasibility of quantum teamwork are the multipartite entangled resources $\varrho_N^{(j)}$ distributed to the parties at each briefing. Above, we assumed different resources be needed at each step j, adapted on the team configuration. This would, however, require some control at the station level, i.e., over the initialization of the protocol, posing a drawback to its versatility. We raise then an intriguing question: Do universal resources exist that enable the unconditional realization of quantum teamwork for *any* arrangement of the two teams? In mathematical terms, do N-partite quantum states ϱ_N exist which are locally equivalent, with respect to any $K|(N-K)$ bipartition, to the tensor product of exactly K maximally entangled states, plus $N - 2K$ uncorrelated single-party states?

Perfect maximally multipartite entangled states for qubits.—Addressing such a question is quite challenging, mainly because the structural aspects of multipartite entanglement are still largely unknown [12]. If one restricts the search for ϱ_N to states of N qubits, however, one has in general a negative answer. Consider, e.g., $N = 4$: While protocols for teleporting an arbitrary state of two qubits via either two Bell pairs [13] or a genuinely four-qubit multipartite entangled state [14] exist, they cannot work for unit fidelity for all of the three possible splittings of four parties into two two-party teams. The most suitable qubit resource states to implement quantum teamwork would be the perfect "maximally multipartite entangled states" (MMES) recently studied by Facchi *et al.* [8], which are those N-qubit states $|\Psi\rangle$ that can be written in Schmidt form as $|\Psi\rangle_{AB} = \sum_{j=1}^{2^K} \sqrt{1/2^K} |\phi_j\rangle_A |\chi_j\rangle_B$ across any $A|B$ partition where the block A is made of K qubits. They have the defining property of exhibiting the maximum allowed bipartite entanglement for all possible splittings. However, it has been proven that qubit perfect MMES do not exist for $N = 4$ and for all $N \geq 8$, indicating the strong presence of an entanglement frustration as complexity increases [8]. Faithful teamwork instances can thus be achieved only among 5 or 6 parties (the case $N = 7$ is still unsettled [8]) when each party is allotted a single qubit.

Perfect maximally multipartite entangled states for continuous variables.—In a CV scenario, one could transliterate the notion of MMES in a weak sense, by looking for states whose bipartite entanglements are simultaneously diverging across all bipartitions. It is easy to find CV states which meet these requirements, the most prominent examples being the GHZ-type (GHZ stands for Greenberger-Horne-Zeilinger) permutation-invariant Gaussian states [11] of N modes, in the limit of infinite local squeezing. However, for any bipartition these states are locally equivalent to a single maximally entangled state [Einstein-Podolsky-Rosen (EPR) pair] and as such enable only a very limiting case of quantum teamwork, where $K = 1$. This special instance of teleporting arbitrary single-mode states from any sender to any receiver, the latter acting

cooperatively with $N - 2$ assisting parties, was already introduced as a "quantum teleportation network" [11] and has been demonstrated experimentally for $N = 3$ [15].

In this Letter, we go beyond these limitations by identifying N-mode CV Gaussian resources which are locally equivalent to a maximal number of EPR pairs for any partition, thus enabling unconditional quantum teamwork, and properly embodying the stronger notion of "perfect" CV MMES. Precisely, we define a perfect CV MMES as a (not necessarily Gaussian) N-mode state such that, with respect to any $A|B$ partition where the block A is made of K modes, it can be transformed by local unitaries into the tensor product of exactly K EPR pairs (and $N - 2K$ uncorrelated single-mode states). Clearly, such a state has infinite energy and stands as an idealized limit; therefore, the notion of perfect CV MMES has to be understood in the sense of a family of states, depending on a parameter (e.g., a local squeezing degree), which converge asymptotically to the ideal perfect CV MMES when the parameter is very large. For any finite value of the parameter, one has an imperfect approximation of the perfect CV MMES, but this approximation can be arbitrarily precise.

We shall look for perfect CV MMES within the family of Gaussian weighted graph states of a N-mode CV system. Gaussian states, up to local displacements, are completely specified by the covariance matrix Γ of the second canonical moments [4]. A weighted graph Gaussian state $|\Psi\rangle$ is a pure N-mode Gaussian state whose covariance matrix is defined in terms of a local squeezing degree $r > 0$ and associated to a weighted N-vertex graph $\mathcal{G}$ with adjacency matrix Ω via a canonical prescription [5–7], as detailed in the supplementary material [16]. For any splitting of the modes into a K-mode block owned by party A and a $(N - K)$-mode block owned by party B (recall that $K \leq N - K$), any pure N-mode Gaussian state $|\Psi\rangle_{AB}$ is equivalent, up to local (with respect to the bipartition) Gaussian unitary operations, to the tensor product of $M \leq K$ entangled two-mode squeezed states $|\psi(r_i)\rangle_{i \in A, i + K \in B}$ and additional $N - 2M$ uncorrelated single-mode vacua [17]. The two-mode squeezed states $|\psi(r)\rangle = \sum_n \lambda^n \sqrt{1 - \lambda^2}|n, n\rangle$ (with $\lambda = \tanh r$) are prototypical CV states which reproduce infinitely entangled EPR pairs in the limit $r \to \infty$. The number M of entangled two-mode squeezed states in the normal form of a pure N-mode Gaussian state for a given partition is given by the symplectic rank of its reduced K-mode covariance matrix [18].

In the case of weighted graph Gaussian states, by writing the adjacency matrix Ω in block form with respect to the $A|B$ partition

$$\Omega = \begin{pmatrix} \Omega_A & \Omega_{AB} \\ \Omega_{AB}^T & \Omega_B \end{pmatrix},$$

one has $M(|\Psi\rangle_{AB}) = \text{rank}(\Omega_{AB})$ [5,19]. Moreover, by construction, for these states, the two-mode squeezed pairs appearing in the normal form all simultaneously converge to EPR pairs for a diverging r. Thus we obtain the following powerful and simple criterion: A pure N-mode

Gaussian weighted graph state with adjacency matrix Ω is a perfect CV MMES in the limit of infinite local squeezing if and only if all of the $K \times (N - K)$ off-diagonal block submatrices Ω_{AB} have matrix rank K.

How quantum teamwork works.—We are ready to provide examples of Gaussian resources useful for quantum teamwork. We first observe that the direct analogues of perfect qubit MMES for $N = 5, 6$ [8], described by unweighted, partially connected graphs, are perfect CV MMES, as one would expect. For $N = 4$, no perfect qubit MMES exist [8]. However, we disclosed several instances of four-mode CV perfect MMES. One such instance is provided by (the infinite-squeezing limit of) a Gaussian state $|\Psi^{(4)}\rangle$ with irreducible three-party correlations which has been recently introduced [20,21] and experimentally demonstrated [22]. It is defined by the action of a beam splitter on two beams each taken from a two-mode squeezed state $|\Psi^{(4)}\rangle = U(t)_{23}[|\psi(r)\rangle_{12} \otimes |\psi(r)\rangle_{34}]$, where $U(t)$ is a beam splitter transformation with transmittivity t. This state is locally equivalent to the four-mode ring weighted CV graph state depicted in Fig. 1(a).

For the resource $|\Psi^{(4)}\rangle$ it is instructive to illustrate the explicit steps needed for the teleportation of arbitrary two-mode states from any two parties paired as team A to the remaining two parties of team B. If $A = (1, 4)$, the only local operation required to obtain 2 two-mode squeezed entangled pairs is obviously an inverse beam splitter with transmittivity t on the B modes. If $A = (1, 2)$, both teams have to locally mix their respective modes at a 50:50 beam splitter and further apply local squeezing operations, with squeezing s on modes 1 and 4 and $1/s$ on modes 2 and 3 $[s = [(\cosh 2r - \sqrt{t} \sinh 2r)/(\cosh 2r - \sqrt{t} \sinh 2r)]^{1/4}]$, to reproduce a dual entangled channel. If $A = (1, 3)$, the same steps are required but with the following modifications: $t \rightarrow 1 - t$ and $2 \leftrightarrow 4$. Suppose now a random arrangement is chosen for the two teams, and team A wishes to teleport an arbitrary two-mode (entangled) state, e.g., a two-mode squeezed state $|\psi(z)\rangle_{\text{in}}$, to team B. After the local operations described above, each input mode is independently teleported according to the conventional Braunstein-Kimble scheme [10,23] using the correspond-

ing two-mode squeezed entangled pair [24], and finally team B obtains the two-mode state ϱ_{out}. The fidelity $\mathcal{F} = {}_{\text{in}}\langle\psi(z)|\varrho_{\text{out}}|\psi(z)\rangle_{\text{in}}$ between input and output reads

$$\mathcal{F} = \frac{\exp(2r_A)}{2(\cosh 2r_A + \cosh 2z)}, \tag{1}$$

depending on both the input squeezing z and the effective squeezing r_A of the two entangled pairs in the normal form of $|\Psi^{(4)}\rangle$ relative to the partition $A|B$. We have $r_{(1,4)} = r$, $r_{(1,2)} = \frac{1}{2}\text{arccosh}\sqrt{\cosh^2 2r - t\sinh^2 2r}$, and $r_{(1,3)} = \frac{1}{2}\text{arccosh}\sqrt{\cosh^2 2r - (1 - t)\sinh^2 2r}$. For any finite z, $0 < t < 1$, and $r \rightarrow \infty$, in which limit the entangled channels reproduce exactly EPR pairs, the fidelity converges to unity for all bipartitions [see Fig. 1(b)]. This is an *unconditional* quantum teamwork realization for $N = 4$, experimentally feasible with current technology [15,22,23].

More examples of perfect CV MMES are provided in the supplementary material [16].

Typicality of continuous variable perfect maximally multipartite entangled states.—We may now attempt an estimate of the volume occupied by perfect CV MMES in the space of pure Gaussian states for arbitrary N. The reduced symplectic eigenvalues $\nu_j^{(A)} \geq 1$ $(j = 1, \ldots, K)$ of the K-mode covariance matrix $\Gamma_A^{(N)}$ of a generic N-mode pure-state covariance matrix Γ_N, corresponding to any block A, are a continuous analytic function of the real elements of the covariance matrix [4]. It follows that the set of states which certainly fail to be perfect MMES, characterized by one or more $\nu_j^{(A)} = 1$, is of null measure. If we specialize to weighted graph Gaussian states, where by construction if all $\nu_j^{(A)} > 1$, then they all diverge for infinite local squeezing, we can conclude that perfect CV MMES families are henceforth *typical* [25] in the space of (weighted graph) pure Gaussian states, in the sense that randomly picked (weighted graph) pure Gaussian states for an arbitrary number of modes are expected to reproduce perfect MMES for diverging local squeezing. We tested the typicality argument by a numerical construction of pure N-mode Gaussian states described by (generally complete) graphs associated to a symmetric adjacency matrix Ω with null principal diagonal and random off-diagonal integer weights $\pm\Omega_{ab} \in [0, N]$. Remarkably, we found instances of perfect CV MMES at the first run for each N (we tested up to $N = 100$). An example for $N = 20$ is presented in the supplementary material [16]. As mentioned, notable exceptions lying in the null-measure set of nonperfect MMES are the CV GHZ-type states [11], associated to fully connected unweighted graphs. As soon as the symmetry is broken and randomness is plugged in the interaction weights, perfect maximal multipartite entanglement blossoms in CV Gaussian states, which are thus unaffected by the entanglement frustration featuring in many-body low-dimensional systems [8].

Applications and remarks on practical implementations.—Perfect CV MMES stand as promising resources
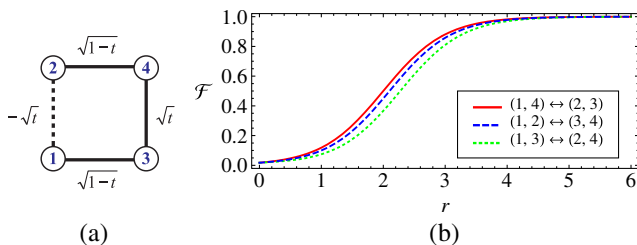

FIG. 1 (color online). (a) Weighted graph definition of four-mode perfect CV MMES locally equivalent to the state $|\Psi^{(4)}\rangle$. (b) Fidelity of the quantum teamwork for teleporting input two-mode squeezed states with squeezing $z = 2$, plotted versus the resource squeezing r for the three possible arrangements of sender and receiver teams (at $t = 1/3$).

for operating the quantum internet [2]. All Gaussian weighted graph states, including perfect CV MMES, are efficiently engineerable with off-line squeezers and linear optics [26]. Let us also recall that several classes of cluster states, belonging to the family of (weighted) graph Gaussian states [6,7], are proven universal resources for CV one-way quantum computation [27]. Given the typicality of perfect CV MMES, we may expect that suitable graph structures exist for arbitrarily large N such to achieve both the universality for quantum computation [28] and the CV MMES property. By construction, perfect CV MMES enable the creation of EPR pairs between any two modes and have unbounded entanglement width scaling linearly with N, thus fulfilling the necessary conditions for efficient universality [28]. Remarkably, perfect CV MMES automatically enable faithful "disk operations" such as cut and paste of large sectors of stored data. Here the input data are encoded in the state of multiple atomic ensembles, team A owns the light modes interacting with the source units, and team B controls the target storage units. Perfect CV MMES for N modes enable, moreover, data compression in the form of multipartite dense coding to transmit N-way classical signals simultaneously [29]. Further applications include quantum secret or state sharing [30].

Clearly, any implementation will be affected by experimental imperfections, mainly traceable to the usage of a finite degree of squeezing in the preparation of the teamwork resources. This results in a non-unit fidelity in the transmission (as seen in Fig. 1(b) for finite r, where e.g., $\mathcal{F} \approx 80\%$ for 20 dB of resource squeezing). The ensuing inefficiency of the protocol propagates with the number K of input modes to be teleported between the two teams. For instance, suppose the teamwork resource is a weighted graph state of $N = 2K$ modes, locally reducible to a tensor product of K two-mode squeezed states, with squeezing r, across any partition into two blocks of K modes each. Let us consider as an input state a K-mode GHZ-type symmetric state with local squeezing z [11,24]. Each mode of the input state is individually teleported via a corresponding approximate EPR channel. The overall teamwork fidelity $\mathcal{F}_K$ scales exponentially with K, as $[(\cosh z)^{-1}\mathcal{F}_1]^K \leq \mathcal{F}_K \leq \mathcal{F}_1{}^K$, where $\mathcal{F}_1 = (1 + \tanh r)/2$ is the fidelity [10] for teleporting a single-mode vacuum via a two-mode squeezed state with squeezing r. We observe that if $r \gg 0$ (quasiperfect CV MMES resource), then the performance is slightly affected by a larger input size, while for moderate (realistic) squeezing r in the resource the performance can be quickly degraded with K, and the degradation is enforced by the amount of entanglement in the input state (in this case, z). Suitable generalizations and optimizations of the conventional Braunstein-Kimble teleportation scheme [10], which is a building block of our networked scheme, might result in improved efficiencies for a given finite squeezing resource.

The demonstration advanced in this Letter of the powerful features of quantum teamwork protocols, peculiar of CV systems and structurally precluded to many-qubit scenarios, opens an avenue for the deep investigation of the characteristic traits of high-dimensional multipartite entanglement and their exploitation to convey multimedia information over a global quantum communication web.

　*jzhang74@yahoo.com; jzhang74@sxu.edu.cn.
 [1] C. H. Bennett and D. P. DiVincenzo, Nature (London) **404**, 247 (2000).
 [2] H. J. Kimble, Nature (London) **453**, 1023 (2008).
 [3] S. L. Braunstein and P. van Loock, Rev. Mod. Phys. **77**, 513 (2005).
 [4] G. Adesso and F. Illuminati, J. Phys. A **40**, 7821 (2007).
 [5] M. Hein et al., Phys. Rev. A **69**, 062311 (2004).
 [6] J. Zhang and S. L. Braunstein, Phys. Rev. A **73**, 032318 (2006).
 [7] J. Zhang, Phys. Rev. A **78**, 052307 (2008).
 [8] P. Facchi et al., Phys. Rev. A **77**, 060304(R) (2008).
 [9] C. H. Bennett et al., Phys. Rev. Lett. **70**, 1895 (1993).
[10] S. L. Braunstein and H. J. Kimble, Phys. Rev. Lett. **80**, 869 (1998).
[11] P. van Loock and S. L. Braunstein, Phys. Rev. Lett. **84**, 3482 (2000).
[12] R. Horodecki et al., Rev. Mod. Phys. **81**, 865 (2009).
[13] G. Gordon and G. Rigolin, Phys. Rev. A **73**, 042309 (2006).
[14] Y. Yeo and W. K. Chua, Phys. Rev. Lett. **96**, 060502 (2006).
[15] H. Yonezawa et al., Nature (London) **431**, 430 (2004).
[16] See EPAPS Document No. E-PRLTAO-103-017935 for supplementary material. For more information on EPAPS, see http://www.aip.org/pubservs/epaps.html.
[17] A. Botero and B. Reznik, Phys. Rev. A **67**, 052311 (2003).
[18] The symplectic rank of a covariance matrix is the number of its symplectic eigenvalues strictly bigger than 1 [4].
[19] D. Fattal et al., arXiv:quant-ph/0406168.
[20] J. Zhang et al., Phys. Rev. A **76**, 064301 (2007).
[21] J. Zhang et al., Phys. Rev. A **77**, 022316 (2008).
[22] A. Tan et al., Phys. Rev. A **78**, 013828 (2008).
[23] A. Furusawa et al., Science **282**, 706 (1998).
[24] G. He et al., J. Phys. B **41**, 215503 (2008).
[25] A. Serafini et al., Phys. Rev. Lett. **98**, 170501 (2007).
[26] P. van Loock et al., Phys. Rev. A **76**, 032321 (2007); N. C. Menicucci et al., Phys. Rev. A **76**, 010302 (2007).
[27] N. C. Menicucci et al., Phys. Rev. Lett. **97**, 110501 (2006).
[28] M. Van den Nest et al., New J. Phys. **9**, 204 (2007).
[29] S. L. Braunstein and H. J. Kimble, Phys. Rev. A **61**, 042302 (2000); X. Y. Li et al., Phys. Rev. Lett. **88**, 047904 (2002).
[30] T. Tyc and B. C. Sanders, Phys. Rev. A **65**, 042310 (2002); A. M. Lance et al., ibid. **71**, 033814 (2005).